Countercyclical Risk Aversion and International Business Cycles

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Abstract

Experimental evidence suggests that agents' risk aversion is time-varying and countercyclical. This evidence is at odds with the conventional practice of a constant risk aversion in DSGE models. Introducing a countercyclical time-varying risk aversion into an otherwise standard international real business cycle model helps to generate data moments that are consistent with empirical observations. By introducing a countercyclical risk aversion we can successfully address the Backus-Smith Puzzle and the International Comovement Puzzle. In addition, introducing a countercyclical time-varying risk aversion helps in generating the correct degree of volatility in investment and labor.

Keywords: Countercyclical Risk Aversion, DSGE, Backus-Smith Puzzle

JEL Classification: F32, F41, F44

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1 Introduction

There is increasing empirical evidence that agents' risk aversion is time-varying and countercyclical. In a recent contribution Cohn et al. (2015) show using a lab experiment with financial professionals that agents become less risk averse in a boom scenario and more risk averse in a bust scenario i.e. risk aversion appears to be countercyclical. In a closely related work Guiso et al. (2018) find countercyclical risk aversion in an experiment with Italian bank customers before and after the financial crises, therefore confirming the findings by Cohn et al. (2015).

At the same time international real business cycle models suffer from various problems or puzzles. Standard international business cycle models predict that (1) The real exchange rate is strongly positively correlated with relative consumption across countries. However, the data shows a correlation of close to zero or even a negative correlation. This anomaly is known in the literature as the Backus-Smith Puzzle.[1] (2) International real business cycle models predict a negative or close to zero correlation of investment and labor across countries. In the data however these correlations are significantly positive, a problem known as the International Comovement Puzzle. (3) Consumption shows a higher correlation across countries than output. In the data however, output is higher correlated than consumption. This problem is known as the Quantity Puzzle.[2] (4) In the model the real exchange rate has a much lower volatility than in the data, a problem widely known as the Price Puzzle.

In this paper we ask the question whether countercyclical risk aversion can help to solve some of these puzzles in international macroeconomics. We do so by introducing time-varying risk aversion into an otherwise standard international real business cycle (RBC) model with incomplete markets. In our reduced form model the risk aversion of agents decreases when output growth turns positive in reaction to a positive transitory total factor productivity (TFP) shock. The subsequent return back to steady state risk aversion increases consumption growth through an additional term in the Euler equation. Since we make both countries' risk aversion dependent on domestic output growth, countercyclical risk aversion alters the relative consumption behavior across countries after a transitory productivity shock. The introduction of a countercyclical risk aversion then leads to a break down of the correlation of the real exchange rate with relative consumption across countries, thus providing a plausible explanation for the Backus-Smith Puzzle. In addition the correlation across countries of investment and labor increases, thus making progress in the International Comovement Puzzle. A positive side effect of our model with countercyclical risk aversion is the additional increase in investment and labor volatility that now matches the empirically observed volatility more closely. The model with countercyclical risk aversion, however, does little to solve the Quantity and Price Puzzles i.e. consumption across countries is still higher correlated than out-

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[1] See Backus and Smith (1993) and Kollmann (1996).

[2] See Backus et al. (1994).

put and the real exchange rate is still not volatile enough. We find that our results are robust across different functional forms for the utility function, namely Cobb-Douglas, Greenwood-Hercowitz-Huffman (GHH), and King-Plosser-Rebelo (KPR) preferences.

This papers combines two areas of economic research. The literature on behavioral economics featuring countercyclical risk aversion on the one hand and the literature on international real business cycle models on the other hand. On the side of the behavioral economics literature, this paper is based on the findings by Cohn et al. (2015) who use a lab experiment to show the existence of countercyclical risk aversion. Since the experiment is conducted in a lab setting it allows for a clear identification of the treatment effect and ensures that the results are not invalidated by measurement error or identification issues. Our paper is also based on the closely related findings by Guiso et al. (2018) who find countercyclical risk aversion in an experiment with Italian bank customers before and after the financial crises. They find that bank customers have a higher risk aversion after the financial crisis in 2009 than the same customers before the crisis in 2007. However, unlike Cohn et al. (2015) it is harder for them to disentangle the effect of a change in the risk aversion from other effects, like wealth effects, on the consumers actions.

There is also a growing literature that incorporates a time-varying risk aversion into macroeconomic and finance models. Among them Brandt and Wang (2003) who consider time-varying risk aversion caused by news about output growth and inflation in a consumption based asset pricing model. As Brandt and Wang (2003) point out, time-variation in the relative risk aversion is widespread in consumption based asset pricing models and can usually be found as habit formation to explain the equity premium puzzle as in Campbell and Cochrane (1999).[3] In a different setting Chu et al. (2014) introduce a state dependent risk aversion into a heterogeneous agent model where the risk aversion depends on the wealth of the household. They find that a model with a state dependent risk aversion predicts a larger wealth inequality than a model with a constant risk aversion. Finally, Benchimol (2014) introduces risk aversion shocks to a New Keynesian model and finds that risk aversion shocks increase inflation and decrease output.

From the perspective of international RBC models our paper is closely related to Mandelman et al. (2011) who introduce investment specific technology (IST) shocks to a two-country model and can break the positive correlation between the real exchange rate and relative consumption. However, after calibrating the model to the US and the rest of the world IST shocks turn out to be not strong enough. When it comes to explain the Backus-Smith Puzzle, our paper is also closely related to Stockman and Tesar (1995) and Heathcote and Perri (2013) who use taste shocks as an explanation. As an alternative explanation for the Backus-Smith Puzzle, Corsetti et al. (2008b) use non-traded goods that can break the correlation between the real exchange rate and relative consumption. By introducing non-traded goods they are also able to address the Price Puzzle of low real exchange rate volatility.

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[3] With preferences of the form $U(C_t) = \frac{(C_t - \tau C_{t-1})^{1-\sigma}}{1-\sigma}$ the relative risk aversion becomes $RRA_t = \sigma \frac{C_t}{C_t - \tau C_{t-1}}$ so that the relative risk aversion depends on the habit stock.

Karabarbounis (2014) uses home production to solve for the Backus-Smith, International Comovement, and Quantity Puzzle. Bai and Rios-Rull (2015) introduce goods market frictions in a model with only demand shocks and are subsequently able to solve the Backus-Smith Puzzle. In addition they successfully address the Quantity Puzzle so that output is more correlated across countries than consumption. Dogan (2019) uses IST shocks originating in the US to explain Mexican business cycles. She finds that due to the high exchange rate volatility of Mexico, investment specific technology shocks in the US get transmitted to Mexico more strongly and are hence sufficient to explain the Backus-Smith and Quantity Puzzle.

Our paper is different from the previous international RBC literature in that we assume that the risk aversion is explicitly time-varying and countercyclical. Our way of introducing a time-varying risk aversion can therefore be seen as a reduced form to implement the empirical observations of countercyclical risk aversion into a standard macroeconomic RBC model. In this way we can introduce the behavioral finding of countercyclical risk aversion into a standard open economy model that does not require investment specific technology shocks like in Mandelman et al. (2011) or taste shocks like in Stockman and Tesar (1995) and Heathcote and Perri (2013) to match the empirically observed moments. We therefore overcome the potential problem of IST shocks not being strong enough or the shortcoming of taste shocks as being hard to empirically measure. Our paper is also different from papers like Benchimol (2014) in the sense that we assume an endogenous process for the risk aversion instead of an exogenous shock. In contrast to papers that look at the effect of a time-varying risk aversion in a consumption based asset pricing model like Brandt and Wang (2003) we look at the implications for international business cycles.

The rest of this paper is organized as follows. The second part will present an international real business cycle model with countercyclical risk aversion and only transitory TFP shocks. The third part will present the results and the fourth part will conclude and provide an outlook for possible future research.

2 Model

The model is a standard two-country open economy incomplete markets model similar to Heathcote and Perri (2002) or Mandelman et al. (2011). The standard international business cycle model is only augmented to include a time-varying countercyclical risk aversion process that depends on final output growth. For simplicity we will show the problem for the home economy, the foreign economy faces exactly the same problem. We follow the convention and indicate foreign variables by a *.

2.1 Households

Representative households maximize life-time utility

$$\max_{C_t, L_t, I_t, K_t, D_t} \quad \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t U\left(C_t, L_t, \sigma_t\right) \right] \tag{1}$$

where $\beta \in (0, 1)$ is the discount factor of the household and $U\left(C_t, L_t, \sigma_t\right)$ is the period utility function, with C_t being consumption and L_t being labor at time t. Finally, σ_t is the time-varying risk aversion of the households that takes the following functional form

$$\sigma_t = \sigma + e^{\gamma_t} - 1 \tag{2}$$

where $\sigma \geq 2$ is the steady state risk aversion and γ_t follows a first-order autoregressive process of the form

$$\gamma_t = \rho_\sigma \gamma_{t-1} - \eta_\sigma \Delta Y_t \tag{3}$$

where ρ_σ is the persistence and η_σ is the elasticity of the risk aversion AR(1) process with respect to final output growth ΔY_t. This functional form of the time-varying risk aversion admits a steady state value for the risk aversion of σ and a lower bound of $\sigma - 1$. By only considering the case of $\sigma \geq 2$ we can avoid the case of logarithmic utility arising when $\sigma_t = 1$. This functional form using final output growth as the driving source of changes in risk aversion is motivated by the findings of Cohn et al. (2015) that risk aversion is time-varying and countercyclical. We explicitly allow for some persistence in the AR(1) process as it appears to be reasonable that agents, when forming their risk aversion, not only consider contemporaneous output growth but most likely also include past values in their decision making.

The capital law of motion for each country is

$$K_t = (1 - \delta) K_{t-1} + I_t - \frac{\phi}{2} I_{t-1} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \tag{4}$$

where K_t denotes capital, I_t denotes investment, and $\delta \in (0, 1)$ is the capital depreciation rate. $\phi > 0$ denotes further a capital adjustment cost to control the speed of investment. Households have the following budget constraint

$$P_t C_t + P_t I_t + P_{H,t} Q_t D_t \leq P_t W_t L_t + P_t R_t K_{t-1} + P_{H,t} D_{t-1} - P_{H,t} \frac{\zeta}{2} D_t^2 \tag{5}$$

where Q_t is the price of the bond D_t. W_t is the wage rate and R_t is the return on capital where both are defined in terms of the final good. Further P_t denotes the CES price aggregate of the home final good as in Equation (A19) and $P_{H,t}$ denotes the price of the home intermediate good, respectively. The last term induces a convex cost of holding bonds which is an increasing function in the bond level to ensure stationarity of the bond level D_t where $\zeta > 0$ is the elasticity of the cost of holding bonds.

2.2 Firms

Final output Y_t is a composite good consisting of the home country and the foreign country intermediate goods produced by the final good producer

$$Y_t = \left[\omega^{\frac{1}{\theta}} Y_{H,t}^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} Y_{F,t}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{6}$$

where $Y_{H,t}$ is the home produced intermediate good, $Y_{F,t}$ is the foreign produced intermediate good, and $\omega \in (0,1)$ is the share of home intermediate goods in total output. Finally, $\theta > 0$ is the elasticity of substitution between home and foreign intermediate goods. The final goods producer in each country then solves the maximization problem

$$\max_{Y_t \geq 0, Y_{H,t} \geq 0, Y_{F,t} \geq 0} \quad P_t Y_t - P_{H,t} Y_{H,t} - P_{F,t} Y_{F,t} \tag{7}$$

subject to the above production function in Equation (6). The intermediate goods producer maximizes profits

$$\max_{L_t \geq 0, K_{t-1} \geq 0} \quad P_{H,t} Y_{H,t} + P_{H,t} Y_{H,t}^* - P_t W_t L_t - P_t R_t K_{t-1} \tag{8}$$

subject to the production function

$$Y_{H,t} + Y_{H,t}^* = e^{A_t} K_{t-1}^{\alpha} L_t^{1-\alpha} \tag{9}$$

where A_t is the home country's productivity and $\alpha \in (0,1)$ is the elasticity of intermediate goods with respect to capital.

2.3 Productivity Shocks

The productivity processes in the two countries follow a VAR(1) process so that for the home country

$$A_t = \rho_A A_{t-1} + \rho_A^* A_{t-1}^* + u_t \tag{10}$$

and similarly in the foreign country

$$A_t^* = \rho_A A_{t-1}^* + \rho_A^* A_{t-1} + u_t^* \tag{11}$$

where ρ_A and ρ_A^* denote the persistence of the TFP process and u_t and u_t^* are the exogenous TFP shocks following

$$u_t \sim \mathcal{N}\left(0, \sigma_u^2\right) \tag{12}$$

$$u_t^* \sim \mathcal{N}\left(0, \sigma_{u^*}^2\right) \tag{13}$$

with mean zero and a known variance σ_u^2 and $\sigma_{u^*}^2$. In this way increases in productivity in one country can slowly spill over to the other country. We only consider transitory TFP shocks in our model as they are sufficient to generate the desired dynamics in the model. However, we verified the results of the model by including permanent shocks to TFP without a change in the general results of our model.

2.4 Market Clearing

Market clearing in each country implies that the final good Y_t in each country is either consumed or used for investment

$$C_t + I_t = Y_t \tag{14}$$

and since bonds are in zero net supply we require that

$$D_t + D_t^* = 0. \tag{15}$$

2.5 Preferences

We consider different utility functions for our two economies to analyze the effect of countercyclical risk aversion on business cycles. We start with a standard Cobb-Douglas form and then present Greenwood-Hercowitz-Huffman (GHH) and King-Plosser-Rebelo (KPR) preferences as two extreme cases of the Jaimovich and Rebelo preferences as in Jaimovich and Rebelo (2009). The form of the utility function and hence the marginal utility of consumption and labor affected by the time-varying risk aversion will enter the model in three ways. The first is via the labor supply decision as shown in Equation (A21). In this case the time-varying risk aversion parameter σ_t simply drops out and does not affect the labor supply directly. Nevertheless, in the case of Cobb-Douglas and KPR preferences where the labor supply depends on consumption, we can observe an increase in the labor volatility when countercyclical risk aversion is present. The second case is the effect of different marginal utilities of consumption in the consumption Euler equation as shown in Equation (A30). We will show later by log-linearizing the consumption Euler equations that, although all of our three preferences are of constant relative risk aversion (CRRA) form when σ_t becomes a constant, they clearly differ in the generated consumption growth when σ_t becomes time-variant.[4] That will be in addition to the already different expressions for consumption growth when the risk aversion is constant. The third effect is via the risk sharing equation as shown in Equation (A29).

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[4] Note that the measure of relative risk aversion $RRA = -\frac{U''(C_t,L_t,\sigma_t)}{U'(C_t,L_t,\sigma_t)} F\left(C_t L_t\right)$, where $F\left(C_t, L_t\right)$ denotes a function that combines consumption and labor in the CRRA function case as $U\left(C_t, L_t, \sigma_t\right) = \frac{F(C_t,L_t)^{1-\sigma_t}}{1-\sigma_t}$, becomes in all our cases $RRA = \sigma_t$.

2.5.1 Cobb-Douglas

As our benchmark we consider Cobb-Douglas preferences of the form

$$U\left(C_t, L_t, \sigma_t\right) = \frac{\left[C_t^{\tau}\left(1-L_t\right)^{1-\tau}\right]^{1-\sigma_t}}{1-\sigma_t} \tag{16}$$

where C_t is consumption and L_t is labor. $\sigma_t > 1$ is the time-varying risk aversion from Equation (2) and $\tau \in (0,1)$ is the consumption share in the utility function. Cobb-Douglas preferences give the following marginal utilities for consumption

$$U_{t,C} = \frac{\tau}{C_t}\left[C_t^{\tau}\left(1-L_t\right)^{1-\tau}\right]^{1-\sigma_t} \tag{17}$$

and for labor

$$U_{t,L} = -\frac{(1-\tau)}{(1-L_t)}\left[C_t^{\tau}\left(1-L_t\right)^{1-\tau}\right]^{1-\sigma_t} \tag{18}$$

and hence labor supply becomes

$$\frac{(1-\tau)}{\tau}\frac{C_t}{(1-L_t)} = W_t. \tag{19}$$

Since the labor supply is in the Cobb-Douglas case dependent on consumption, changes in the consumption growth rate introduced by the time-varying risk aversion will directly affect labor supply.

2.5.2 GHH

We then consider GHH preferences as in Greenwood et al. (1988) that are known to have no wealth effect and labor supply only depends on wages

$$U\left(C_t, L_t, \sigma_t\right) = \frac{\left(C_t - \psi L_t^{\nu}\right)^{1-\sigma_t}}{1-\sigma_t} \tag{20}$$

where $\psi > 0$ and $\nu > 0$ with the following marginal utilities for consumption

$$U_{t,C} = \left(C_t - \psi L_t^{\nu}\right)^{-\sigma_t} \tag{21}$$

and for labor

$$U_{t,L} = -\left(C_t - \psi L_t^{\nu}\right)^{-\sigma_t}\left(\nu\psi L_t^{\nu-1}\right) \tag{22}$$

and hence labor supply becomes

$$\nu\psi L_t^{\nu-1} = W_t. \tag{23}$$

Since the labor supply is independent of both the time-varying risk aversion σ_t, and in the GHH case also of consumption, the behavior of labor will be the same for a constant risk aversion and a time-varying risk aversion.

2.5.3 KPR

We then analyze KPR preferences as in King et al. (1988) of the form

$$U\left(C_t, L_t, \sigma_t\right) = \frac{\left[C_t\left(1 - \psi L_t^{\nu}\right)\right]^{1-\sigma_t}}{1 - \sigma_t} \tag{24}$$

where $\psi > 0$ and $\nu > 0$ with the following marginal utilities for consumption

$$U_{t,C} = \left[C_t\left(1 - \psi L_t^{\nu}\right)\right]^{-\sigma_t}\left(1 - \psi L_t^{\nu}\right) \tag{25}$$

and for labor

$$U_{t,L} = -\left[C_t\left(1 - \psi L_t^{\nu}\right)\right]^{-\sigma_t}\left(\nu\psi C_t L_t^{\nu-1}\right) \tag{26}$$

and hence labor supply becomes

$$\frac{\nu\psi C_t L_t^{\nu-1}}{1 - \psi L_t^{\nu}} = W_t. \tag{27}$$

Similar to the Cobb-Douglas case in Equation (19), labor supply depends on consumption and hence faster consumption growth caused by a change in the risk aversion will be reflected in higher growth in the labor supply.

2.5.4 Preference Parameters

For all preferences we calibrate the labor elasticity of supply such that in steady state the labor supply equals 0.30 so that we can compare our results across preference specification. For the Cobb-Douglas preferences we get using the labor supply equation in Equation (19) and by solving for the Cobb-Douglas consumption share τ that[5]

$$\tau = \frac{L\frac{C}{Y}}{L\frac{C}{Y} + (1 - L)(1 - \alpha)} \tag{28}$$

where L is the exogenously fixed steady state labor supply. $\frac{C}{Y}$ denotes the implied steady state consumption to output ratio dependent on parameters α,β, and δ.[6] Given a standard parameterization this implies a value of about 0.33 for τ.[7] Following Raffo (2008) and

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[5] See Appendix A.4 for a derivation.

[6] Note that the consumption to output ratio can easily be expressed dependent on parameters as $\frac{C}{Y} = 1 - \frac{\alpha\beta\delta}{1-\beta-\beta\delta}$.

[7] See Table 1 for all parameter values of the model.

Mandelman et al. (2011) we derive the two parameters of the GHH and KPR preferences by imposing that the Frisch labor supply elasticities ϵ are equal across preferences such that

$$\epsilon_{CD} = \epsilon_{GHH} = \epsilon_{KPR}. \tag{29}$$

We continue by using that the Frisch labor supply elasticity in the Cobb-Douglas case can be defined as[8]

$$\epsilon_{CD} = \frac{(1-L)\left(1-\tau\left(1-\sigma\right)\right)}{\sigma L} \tag{30}$$

where σ denotes the steady state risk aversion.[9] Similarly, in the GHH case one gets for the Frisch labor supply elasticity

$$\epsilon_{GHH} = \frac{1}{\nu_{GHH}-1} \tag{31}$$

so that we can calculate ν_{GHH} as

$$\nu_{GHH} = \frac{1+\epsilon_{GHH}}{\epsilon_{GHH}} \tag{32}$$

and using the labor supply equation in Equation (23) the parameter ψ_{GHH} follows as

$$\psi_{GHH} = \frac{WL^{1-\nu_{GHH}}}{\nu_{GHH}} \tag{33}$$

where W is the steady state wage. For the KPR utility we follow Holden et al. (2018) who derive the Frisch labor supply elasticity as

$$\epsilon_{KPR} = \left[\nu - 1 + \frac{\nu\psi L^{\nu}\left(2\sigma-1\right)}{\sigma\left(1-\psi L^{\nu}\right)}\right]^{-1} \tag{34}$$

together with the labor supply equation in Equation (27)

$$\nu\psi C L^{\nu-1} = W\left(1-\psi L^{\nu}\right) \tag{35}$$

we can solve for the two unknown parameters ν_{KPR} and ψ_{KPR} in the KPR case. So that

$$\nu_{KPR} = \frac{1+\epsilon_{KPR}}{\epsilon_{KPR}} - \frac{WL\left(2\sigma-1\right)}{\sigma C} \tag{36}$$

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[8] See e.g. Mandelman et al. (2011).

[9] In what follows, we denote steady state values of a variable by referring to that variable without a time subscript.

where C is steady state consumption. And finally

$$\psi_{KPR} = \frac{W}{\nu_{KPR} C L^{\nu_{KPR}-1} + W L^{\nu_{KPR}}}. \tag{37}$$

2.6 Equilibrium and Equilibrium Conditions

An equilibrium of this economy is characterized by a set of allocations for the consumer in each country consisting of consumption C_t, labor L_t, capital K_t, investment I_t, and bond D_t. The allocations for home and foreign intermediate goods producers $Y_{H,t}$, $Y^*_{H,t}$, $Y_{F,t}$, and $Y^*_{F,t}$, the allocations for both the home and foreign goods producers Y_t and Y^*_t and prices of the intermediate goods $P_{H,t}$, $P^*_{H,t}$, $P_{F,t}$, and $P^*_{F,t}$ as well as final prices P_t and P^*_t. Finally, we require the price of labor and capital W_t and R_t for each country and the price of the bond Q_t as well as the risk aversion for each country σ_t and σ^*_t such that (1) Households allocations solve the households' problem. (2) Intermediate goods producers' allocations solve the intermediate goods producers' problem. (3) Final good producers' allocation solve the final goods producers' problem. (4) All markets clear.

2.7 Solution Method, Steady State, and Equilibrium Conditions

To solve the model we use a first-order approximation around the steady state. We also approximate the model to second and third-order to verify our results. The obtained second moments of the model, however, show no noticeable difference between a first, second, or third-order approximation. We provide the steady states for all variables and a numerical example for the steady state in the Appendix A.1. The full set of equilibrium conditions can be found in the Appendix A.2.

2.8 Parameters

The model is parameterized as in Mandelman et al. (2011) to replicate the quarterly data for the US and the rest of the world. Both countries are assumed to be fully symmetric. All parameters of the model can be found in Table 1. The capital elasticity α is set to 0.36 and the discount factor β is set to 0.99 to match the quarterly empirical data moments. We set the depreciation rate δ to 0.025 and the steady state risk aversion σ to 2.00, respectively. The elasticity of substitution between the home and foreign intermediate goods θ is set to 0.62 which is consistent with the value used by Raffo (2010) but slightly higher than the value used by Corsetti et al. (2008a) and lower than the value used by Heathcote and Perri (2002).[10] The capital adjustment cost ϕ is set to 0.60. The share of home intermediate goods in the final good production function ω is set to 0.90 which matches the home bias found in the data. We assume further a low value of 0.001 for the cost of holding bonds ζ to ensure stationarity of the model. Following Mandelman et al. (2011) and their

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[10] Since the elasticity of substitution between home and foreign intermediate goods is crucial for the cross-country dynamics of the model we will provide robustness checks with the higher value of 0.85 used by Heathcote and Perri (2002).

empirical findings for the US and the rest of the world we assume a correlation of the TFP innovations u_t and u_t^* of 0.29. The persistence of the home and foreign component of the TFP process ρ_A and ρ_A^* is set to 0.97 and 0.025. It thereby matches the empirically observed values by Mandelman et al. (2011) for the US and the rest of the world.[11]

When we fix the labor supply in the steady state L to 0.30 we obtain the following parameters in Table 1 for the three different utility functions. The Cobb-Douglas consumption share τ is calculated as 0.33 and the Frisch labor supply elasticity ϵ is 1.55 for all utility functions. In the case of GHH preferences we get that ν_{GHH} is 1.64 and ψ_{GHH} is 3.13. For the KPR preferences we obtain ν_{KPR} as 0.35 and ψ_{KPR} as 1.08.

Since it is difficult to come up with reasonable estimates of the parameters of the AR(1) process ρ_σ and η_σ for the risk aversion we estimate these two parameters using SMM to match the second moments of the model and the empirical data moments. We target the standard deviation of output to obtain reasonable values for the mean volatility of our model. Since we know that our model has some problems matching the relative standard deviation of consumption and output and the correlation between output and consumption we include these two moments in our SMM routine. We also include the correlation of the real exchange rate and relative consumption across countries and the correlation of output, consumption, investment, and labor across countries.[12] In this way we try to match eight moments with two parameters.

TABLE 1
Model Parameters

Parameter	Description	Value
α	Capital Elasticity	0.36
β	Discount Factor	0.99
δ	Capital Depreciation Rate	0.025
σ	Steady State Risk Aversion	2.00
θ	Elasticity of Substitutability between Intermediate Goods	0.62
ϕ	Capital Adjustment Cost	0.60
ω	Share of Home Intermediate Goods - Home Bias	0.90
ζ	Cost of Bond Holding	0.001
ρ_A	TFP Persistence Home Component	0.97
ρ_A^*	TFP Persistence Foreign Component	0.025
σ_u^2	TFP Shock Variance Home	0.01
$\sigma_{u^*}^2$	TFP Shock Variance Foreign	0.01
L	Steady State Labor Supply	0.30
τ	Cobb-Douglas Consumption Share	0.33
ϵ	Frisch Labor Supply Elasticity	1.55
ν_{GHH}	GHH Utility Parameter	1.64
ψ_{GHH}	GHH Utility Parameter	3.13
ν_{KPR}	KPR Utility Parameter	0.35
ψ_{KPR}	KPR Utility Parameter	1.08

Note: Table 1 shows the parameter values in the DSGE model.

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[11] For robustness we will also try a correlation of the TFP shocks of zero and no spillovers of the TFP shocks across countries.

[12] Not including these cross-country moments in the SMM routine would lead to a case where the SMM predicts a constant risk aversion. However, we believe that a potential small loss in the standard deviation of consumption is worth accepting given the substantial gains in the cross-country moments we can achieve.

3 Results

We evaluate the performance of our model with countercyclical risk aversion using the second moments of the model. We linearize the model around the steady state and generate data series of 500 periods and subsequently discard the first 364 periods as a burn-in to get rid of initial conditions. This leaves us with 136 periods, the same number of periods as in the empirical data provided by Mandelman et al. (2011) which we use for comparison. We replicate the model 500 times using a different sequence of error terms and calculate the median moment and the 5th and 95th percent confidence bands of the moments. All data is in logarithms and HP-Filtered as in Hodrick and Prescott (1997) using a filter weight of 1600 to match the quarterly frequency of the empirical data. We use the empirical data presented by Mandelman et al. (2011) for comparison which ranges from 1973Q1 to 2006Q4 and represents the US and the rest of the world, where the rest of the world consists of the Euro area, the United Kingdom, Canada, Japan, and Australia.

3.1 Moments

Table 2 shows the second moments for different preference functions. The column "Constant RA" for each preference function shows the second moments when η_σ is set to zero and hence the risk aversion σ_t becomes a constant parameter σ with value 2.00. The column "Variable RA" shows the second moments when the two risk aversion parameters ρ_σ and η_σ are estimated using SMM and the eight moments described in the parameter section.[13]

3.1.1 Cobb-Douglas

When we are targeting the moments of the model described in the parameter section using our two risk aversion parameters ρ_σ and η_σ we obtain estimates for ρ_σ of about 0.80 and for η_σ of about 3.54 as can be seen in column (2) of Table 2. Once we add a countercyclical risk aversion to the model using a Cobb-Douglas utility function as in Equation (16) we observe that output, investment, and labor become more volatile. This is especially true for investment and labor where the relative standard deviation to output is now close to the empirical value and almost twice as large as in a model with a constant risk aversion. For the relative consumption to output volatility we observe, however, a decrease compared to a model with a constant risk aversion. The correlation of the real exchange rate and relative consumption across countries becomes virtually zero with a value of 0.11 and now matches the empirical value of -0.04 closely, thereby solving the Backus-Smith Puzzle. In addition, we can also observe that the comovement of investment and labor across countries increases and is now closer to the empirical data. We

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[13] We minimize the sum of squared percentage deviations rather than the sum of squared residuals as our targeted moments are in different units and sizes. The main results stay the same when we minimize the sum of squared residuals.

are therefore able to make some progress in the International Comovement Puzzle. Using Cobb-Douglas preferences in combination with a countercyclical risk aversion leads however to a decline in the correlation of output and consumption due to a change in the reaction of consumption to a TFP shock. Interestingly, the introduction of a countercyclical risk aversion has no effect on the relative volatility of the real exchange rate and consumption, and on the autocorrelation of the real exchange rate. In both cases the real exchange rate does not show enough volatility and is not persistent enough. We thereby fail to make any grounds on the Price Puzzle. Similarly to a constant risk aversion model, our model fails to generate a higher correlation of output across countries than consumption correlation. We are therefore not able to solve the Quantity Puzzle.

3.1.2 GHH

Using a GHH preference specification as in Equation (20) again allows us to generate a correlation of the real exchange rate and relative consumption across countries which is close to zero with 0.10 as can be seen in column (4) of Table 2. The estimated risk aversion parameters η_σ is close to its counterpart in the Cobb-Douglas case with an estimated value of 4.40. However, the persistence parameter ρ_σ is now lower with a value of 0.55. Contrary to the Cobb-Douglas preferences the model is still able to produce a high correlation of output and consumption of 0.80 compared to an almost perfect correlation in the GHH model with a constant risk aversion and a value of 0.84 in the data. Similar to Cobb-Douglas preferences we see a significant increase in the relative volatility of investment and an, albeit, smaller decrease in the relative volatility of consumption to output. Since the labor supply in the GHH case is independent of both, the time-varying risk aversion and consumption, the relative standard deviation of labor to output is unchanged when we introduce time-variation in the risk aversion.

3.1.3 KPR

By using a KPR utility function as in Equation (24) gives a value of -0.05 for the correlation of the real exchange rate and relative consumption as can be seen in column (6) of Table 2. In addition, the cross-country correlations for investment and labor turn positive when the risk aversion becomes countercyclical with values of 0.18 and 0.26, respectively. Although these cross-country correlations are still below the empirically observed value, they mark a significant improvement over the model with a constant risk aversion which shows cross-country correlations for investment and labor of -0.15 and -0.26, respectively. Similar to Cobb-Douglas and GHH preferences, the introduction of a countercyclical risk aversion has only a small effect on the relative volatility of the real exchange rate and output which remains well below the empirically observed values or on the autocorrelation of the real exchange rate. However, contrary to Cobb-Douglas and GHH preferences the estimated risk aversion parameter ρ_σ and η_σ are significantly lower with values of 0.41 and 1.92, respectively.

TABLE 2
Second Moments

		Cobb-Douglas		GHH		KPR	
		(1)	(2)	(3)	(4)	(5)	(6)
	Data	Constant RA	Variable RA	Constant RA	Variable RA	Constant RA	Variable RA
ρ_σ		0.00	0.80	0.00	0.55	0.00	0.41
η_σ		0.00	3.54	0.00	4.00	0.00	1.92
$\sigma(Y)$	1.58	1.37	1.82	1.88	1.90	1.36	1.75
		(1.11;1.71)	(1.43;2.30)	(1.53;2.31)	(1.53;2.37)	(1.10;1.68)	(1.44;2.10)
$\sigma(C)/\sigma(Y)$	0.76	0.68	0.36	0.85	0.62	0.72	0.49
		(0.63;0.74)	(0.31;0.44)	(0.81;0.91)	(0.54;0.70)	(0.67;0.79)	(0.42;0.56)
$\sigma(I)/\sigma(Y)$	4.55	2.14	3.49	1.55	2.69	2.00	3.10
		(1.98;2.28)	(3.31;3.66)	(1.41;1.69)	(2.45;2.92)	(1.83;2.17)	(2.89;3.30)
$\sigma(L)/\sigma(Y)$	0.75	0.25	0.57	0.59	0.59	0.25	0.58
		(0.23;0.27)	(0.54;0.60)	(0.57;0.60)	(0.58;0.60)	(0.22;0.28)	(0.53;0.63)
$\sigma(RER)/\sigma(Y)$	3.06	1.38	1.39	1.29	1.52	1.57	1.77
		(1.09;1.72)	(1.10;1.77)	(1.02;1.60)	(1.19;1.87)	(1.25;1.96)	(1.46;2.14)
$\rho(Y,L)$	0.87	0.85	0.94	1.00	1.00	0.85	0.88
		(0.77;0.90)	(0.91;0.95)	(1.00;1.00)	(1.00;1.00)	(0.77;0.90)	(0.85;0.91)
$\rho(Y,C)$	0.84	0.94	0.51	0.98	0.80	0.95	0.69
		(0.92;0.96)	(0.37;0.62)	(0.97;0.99)	(0.73;0.86)	(0.93;0.97)	(0.59;0.77)
$\rho(Y,I)$	0.91	0.95	0.97	0.95	0.92	0.95	0.95
		(0.93;0.97)	(0.95;0.98)	(0.92;0.97)	(0.89;0.94)	(0.92;0.97)	(0.93;0.96)
$\rho(RER)$	0.82	0.61	0.61	0.54	0.49	0.59	0.47
		(0.48;0.71)	(0.48;0.70)	(0.40;0.66)	(0.36;0.61)	(0.46;0.70)	(0.35;0.57)
$\rho(RER,C/C^*)$	-0.04	0.99	0.11	0.98	0.10	1.00	-0.05
		(0.99;0.99)	(-0.08;0.30)	(0.97;0.98)	(-0.08;0.28)	(0.99;1.00)	(-0.26;0.15)
$\rho(Y,Y^*)$	0.44	0.38	0.40	0.55	0.61	0.36	0.42
		(0.08;0.61)	(0.09;0.63)	(0.31;0.72)	(0.37;0.77)	(0.06;0.60)	(0.16;0.61)
$\rho(C,C^*)$	0.36	0.68	0.63	0.81	0.81	0.75	0.69
		(0.54;0.79)	(0.38;0.79)	(0.69;0.88)	(0.61;0.91)	(0.61;0.84)	(0.45;0.84)
$\rho(I,I^*)$	0.28	0.03	0.19	0.00	0.35	-0.15	0.18
		(-0.32;0.36)	(-0.17;0.48)	(-0.31;0.31)	(0.08;0.55)	(-0.46;0.19)	(-0.06;0.38)
$\rho(L,L^*)$	0.40	0.09	0.28	0.67	0.71	-0.26	0.26
		(-0.28;0.43)	(-0.09;0.55)	(0.47;0.80)	(0.51;0.83)	(-0.55;0.09)	(0.03;0.43)

Note: Table 2 shows the moments of the model. σ denotes the standard deviation of a variable and ρ denotes the correlation between two variables. 5th and 95th percentile confidence bands in parenthesis. Moments are the median of 500 replications.

3.2 Impulse Responses

Figure 1 and Figure 2 show the impulse responses of a one percent transitory TFP shock in the home country for the case of Cobb-Douglas preferences.[14] There are two striking features of the impulse responses generated by the model with countercyclical risk aversion denoted by the dashed-dotted red line. First, the impulse response of all variables becomes more pronounced. And second, the impulse response for relative consumption shown in Figure 2 changes the shape. This change in the behavior of relative consumption is caused by the large increase in consumption in the home country in response to a technology shock, whereas the foreign country's consumption behavior changes only slightly. Examining the impulse response of consumption in the home country more closely reveals that consumption initially reacts less on impact but is subsequently subject to a higher growth rate. Together with the observed increase in output volatility this

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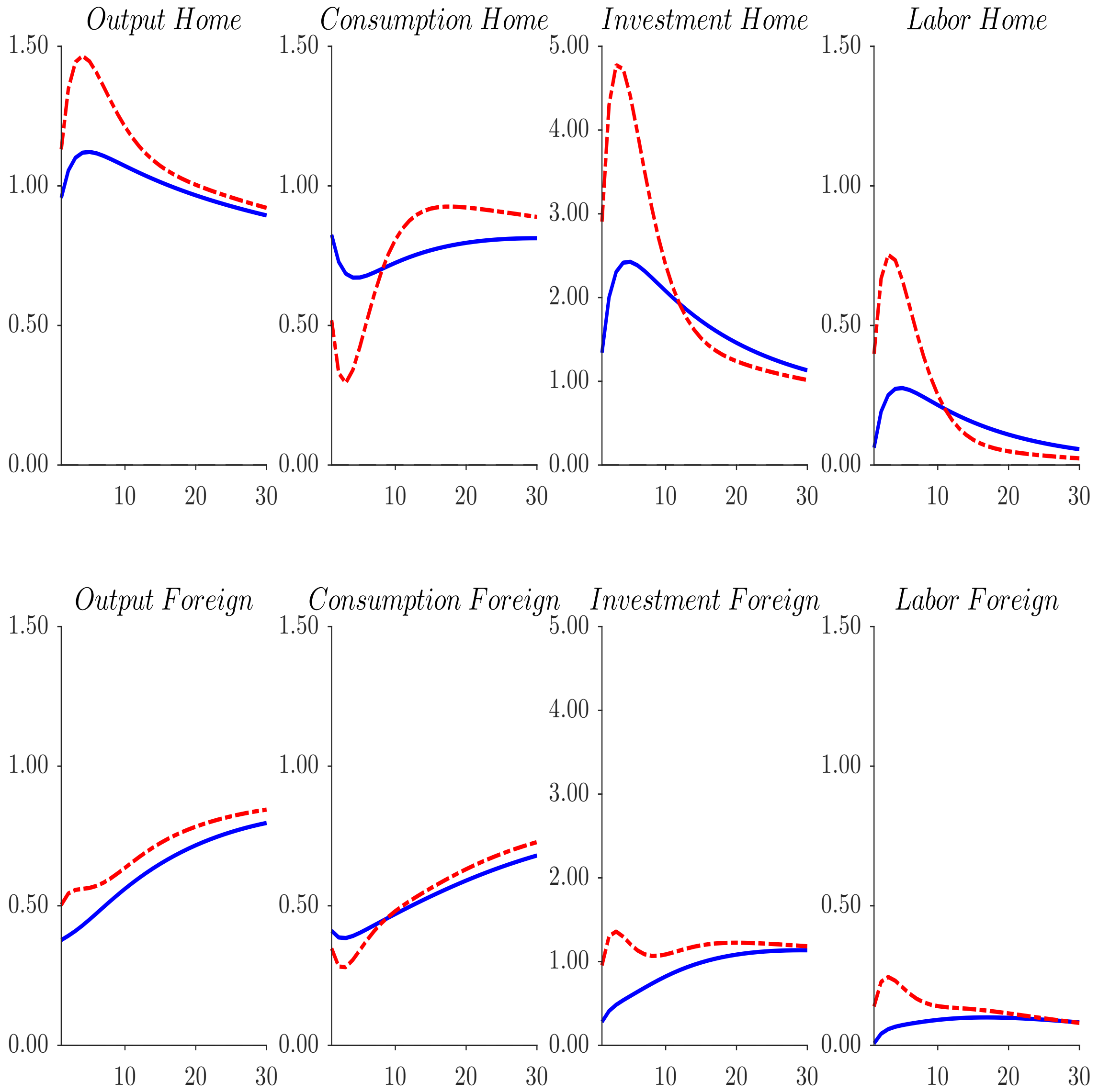
[14] Although our TFP shock is only transitory, it is very persistent when applying the same parameters as Mandelman et al. (2011) and impulse responses consequently converge only slowly back to the steady state. Our main results stay the same once we use a less persistent process for TFP as we show in the robustness section.

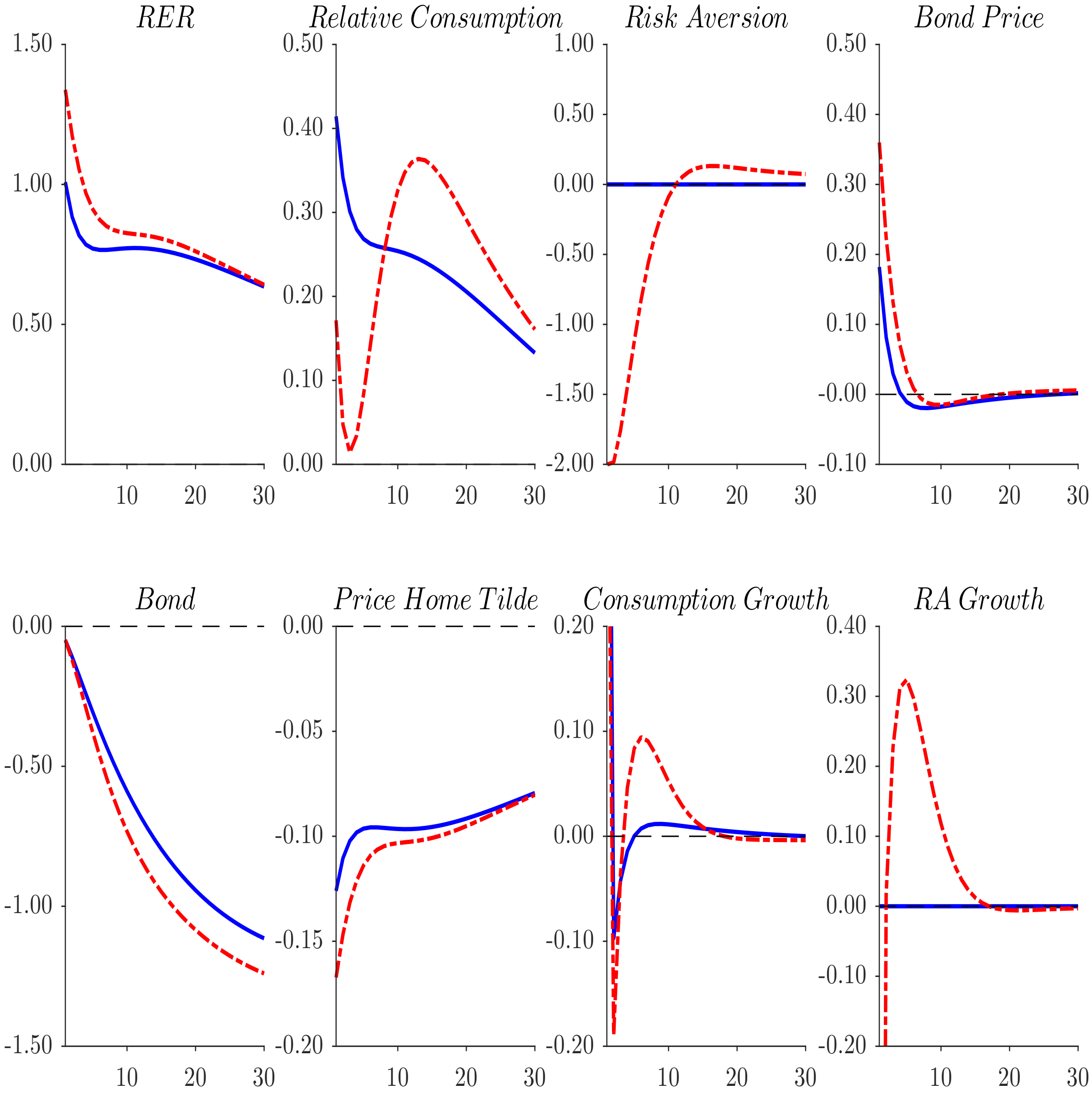
leads to a lower relative volatility of consumption to output as is evident in column (2) of Table 2. From Figure 2 it becomes also clear that the movements in the risk aversion σ_t induced by a home technology shock are rather small. A one percent TFP shock decreases risk aversion by about 2.0 percent from its steady state value of 2.00. A rather small change compared to the wide range of estimates for risk aversion parameters in macroeconomic models that usually include the case of log utility i.e. a risk aversion of 1 and substantially higher values in many asset pricing models. Figure 2 also shows clearly that the growth rate in the risk aversion and consumption growth in the home country share the same pattern after a couple of periods.

FIGURE 1
Impulse Responses I



Note: Figure 1 shows the impulse responses of home and foreign output, consumption, investment, and labor to a home country TFP shock using Cobb-Douglas preferences. Model with a constant risk aversion in blue and with time-varying risk aversion as the dashed-dotted red line. Scale is in percentage deviations from the steady state.

FIGURE 2
Impulse Responses II



Note: Figure 2 shows the impulse responses of additional variables to a TFP shock in the home country using Cobb-Douglas preferences. Model with a constant risk aversion in blue and with time-varying risk aversion as the dashed-dotted red line. Scale is in percentage deviations from the steady state.

3.3 Intuition

We now investigate why the consumption impulse response is more pronounced when the risk aversion becomes countercyclical and why the labor supply reacts more volatile to a TFP shock in that case when we use Cobb-Douglas or KPR preferences. In short, we find that the introduction of a time-varying and countercyclical risk aversion leads to an additional term in the log-linearized Euler equation that leads to higher consumption growth in response to a positive productivity shock. Since labor supply is directly dependent on consumption in the Cobb-Douglas and KPR case we can observe an amplification of the labor response after a productivity shock.

3.3.1 Log-Linear Euler Equation

The Euler equation of the economy is given by

$$\frac{Q_t + \zeta D_t}{\beta} = \mathbb{E}_t \left[\frac{U_{t+1,C}}{U_{t,C}} \frac{\tilde{P}_{H,t+1}}{\tilde{P}_{H,t}} \right] \tag{38}$$

where $\tilde{P}_{H,t}$ is the relative price of the home intermediate good to the final good and $U_{t,C}$ and $U_{t+1,C}$ denote marginal utility of consumption. Then for the case of Cobb-Douglas preferences we get

$$\frac{Q_t + \zeta D_t}{\beta} = \mathbb{E}_t \left[\frac{C_t}{C_{t+1}} \frac{\left[C_{t+1}^{\tau} \left(1 - L_{t+1}\right)^{1-\tau} \right]^{1-\sigma_{t+1}}}{\left[C_t^{\tau} \left(1 - L_t\right)^{1-\tau} \right]^{1-\sigma_t}} \frac{\tilde{P}_{H,t+1}}{\tilde{P}_{H,t}} \right] \tag{39}$$

note that the risk aversion σ_t is now dependent on time t in the Euler equation. Taking logarithms of the Euler equation gives

$$\begin{aligned} &\ln\left(Q_t + \zeta D_t\right) - \ln\left(\beta\right) + \ln\left(\tilde{P}_{H,t}\right) - \ln\left(\tilde{P}_{H,t+1}\right) = \ln\left(C_t\right) - \ln\left(C_{t+1}\right) + \left(1 - \sigma_{t+1}\right) \tau \ln\left(C_{t+1}\right) \\ &+ \left(1 - \sigma_{t+1}\right)\left(1 - \tau\right) \ln\left(1 - L_{t+1}\right) - \left(1 - \sigma_t\right) \tau \ln\left(C_t\right) - \left(1 - \sigma_t\right)\left(1 - \tau\right) \ln\left(1 - L_t\right) \end{aligned} \tag{40}$$

the log-linearization around the steady state follows then after canceling out redundant terms as

$$\begin{aligned} &-\widetilde{r}_t + \zeta \widetilde{D}_t + \widetilde{P_{H,t}} - \widetilde{P_{H,t+1}} = \widetilde{C}_t \left[1 - \tau + \sigma\tau\right] - \widetilde{C_{t+1}} \left[1 - \tau + \sigma\tau\right] - \widetilde{L}_t \frac{L}{1 - L} \left[\left(\sigma - 1\right)\left(1 - \tau\right)\right] \\ &+ \widetilde{L_{t+1}} \frac{1}{1 - L} \left[\left(\sigma - 1\right)\left(1 - \tau\right)\right] - \tau \ln\left(C\right)\left(\sigma_{t+1} - \sigma_t\right) - \left(1 - \tau\right) \ln\left(1 - L\right)\left(\sigma_{t+1} - \sigma_t\right) \end{aligned} \tag{41}$$

where as usual $\widetilde{x}_t = \frac{x_t - x}{x}$ denotes percentage deviations from the steady state of a variable. Then after rearranging terms we get for consumption growth

$$\widetilde{C_{t+1}} - \widetilde{C}_t = \frac{\widetilde{r}_t - \zeta \widetilde{D}_t - \widetilde{P_{H,t+1}} + \widetilde{P_{H,t}}}{1 - \tau + \sigma\tau} + \Gamma \left(\widetilde{L_{t+1}} - \widetilde{L}_t \right) + \Lambda \left(\widetilde{\sigma_{t+1}} - \widetilde{\sigma}_t \right) \tag{42}$$

where

$$\Gamma = \frac{L}{1-L}\left[\frac{(\sigma-1)(1-\tau)}{1-\tau+\sigma\tau}\right] \tag{43}$$

and Λ can be considered as the elasticity of consumption growth with respect to growth in the risk aversion and equals

$$\Lambda = -\left[\frac{\tau\ln(C)+(1-\tau)\ln(1-L)}{1-\tau+\sigma\tau}\right]. \tag{44}$$

Compared to a model with a constant risk aversion, we have an additional term that affects consumption growth, namely the growth rate of the risk aversion. It is worth noting that time variation in the risk aversion also has general equilibrium effects on the behavior of the interest rate $\widetilde{r_t}$, the bond $\widetilde{D_t}$, and the price of home intermediate goods relative to final goods $\widetilde{\tilde{P}_{H,t}}$ in the Euler equation.[15] However, a look that the impulse responses suggests that these effects are rather small and possibly negligible. We therefore assume that changes in consumption growth are caused by the additional term in the log-linearized Euler equation in Equation (42).

After a positive TFP shock output growth increases and hence the risk aversion decreases suddenly as can be seen in Figure 2. After the initial decline in the risk aversion σ_t returns to its steady state value and shows a positive growth rate. Agents will react by increased consumption growth and decreasing labor supply while the risk aversion returns back to its steady state. Hence roughly between period 2 and 20 we observe positive consumption growth and a quick return of labor supply back to its steady state value. However, as the risk aversion increases and finally reaches its steady state, consumption growth will decline and return back to steady state.

As both countries' risk aversion is dependent on the growth rate of the final output of each country, the home country's consumption will show a more pronounced impulse response than the foreign country's consumption in response to a home productivity shock, which remains basically unchanged. This creates a relative consumption impulse response which is driven by the home country's impulse response pattern. As the impulse response of the real exchange rate remains almost unaffected by the introduction of a countercyclical risk aversion, the correlation of the real exchange rate and relative consumption breaks down.

In our model the initial drop in consumption further leads to an investment boom which leads to higher capital and hence a higher marginal return of labor i.e. wages increase and through the labor supply equation labor increases more than in the case with a constant risk aversion. We provide the log-linearized Euler equation for the case of GHH and KPR preferences in the Appendix A.3. The log-linearized Euler equation in these cases follows the same pattern and includes an additional term that accounts for

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[15] Note that $\tilde{P}_{H,t}$ refers to the relative price of the home intermediate good to the final good. Whereas $\widetilde{\tilde{P}_{H,t}}$ refers to the percentage deviation of that variable from the steady state.

the growth rate of the risk aversion.

3.3.2 Euler Equation Elasticities

Table 3 shows the Euler equation elasticities Γ and Λ for the parameters used in Table 1 and a steady state labor supply L of 0.30 for Cobb-Douglas, GHH, and KPR preferences. Setting these values into context of the impulse responses in Figure 2 suggests in the Cobb-Douglas case that at its peak 0.10 percent growth in the risk aversion directly causes consumption growth to be higher by 0.023 percent between two periods. The remaining portion of higher consumption growth in the presence of countercyclical risk aversion is explained by the change in the remaining terms in the Euler equation when countercyclical risk aversion is present.

To verify the sign of the elasticity of consumption growth with respect to growth in the risk aversion Λ we calculate the steady state values of consumption C and the Cobb-Douglas consumption share τ given a grid of values for steady state labor L.[16] We further test for robustness and draw 10000 values for α, β, and δ that affect steady state consumption C and the consumption share τ and therefore the size and sign of the elasticity Λ from a random uniform distribution.[17] Figure 3 shows the size of the elasticity of consumption growth with respect to growth in the risk aversion for different steady state values of labor for our three preference function specifications. We show the median, as well as the 5th and 95th percentile confidence bands of all calculated values for Λ. The simulations imply two outcomes. First, the elasticity of consumption growth with respect to growth in the risk aversion is positive for all three specifications for reasonable parameterizations of the model. Second, the elasticity of Cobb-Douglas preferences is the smallest, followed by GHH preferences and KPR preferences that imply the largest risk aversion elasticity. The relative size of the estimated elasticities are in line with the size of elasticities estimated for the risk aversion process. Specifically, the estimated parameter η_σ is lower in the KPR case than in the Cobb-Douglas or GHH case as the consumption Euler equation already features a higher multiplier Λ and therefore smaller changes in the risk aversion are required.

TABLE 3
Euler Equation Elasticities

Parameter	Description	Cobb-Douglas	GHH	KPR
Γ	Labor Growth Elasticity	0.21	0.86	0.43
Λ	Risk Aversion Growth Elasticity	0.23	0.44	1.43

Note: Table 3 shows the implied Euler equation elasticities using the DSGE parameters shown in Table 1.

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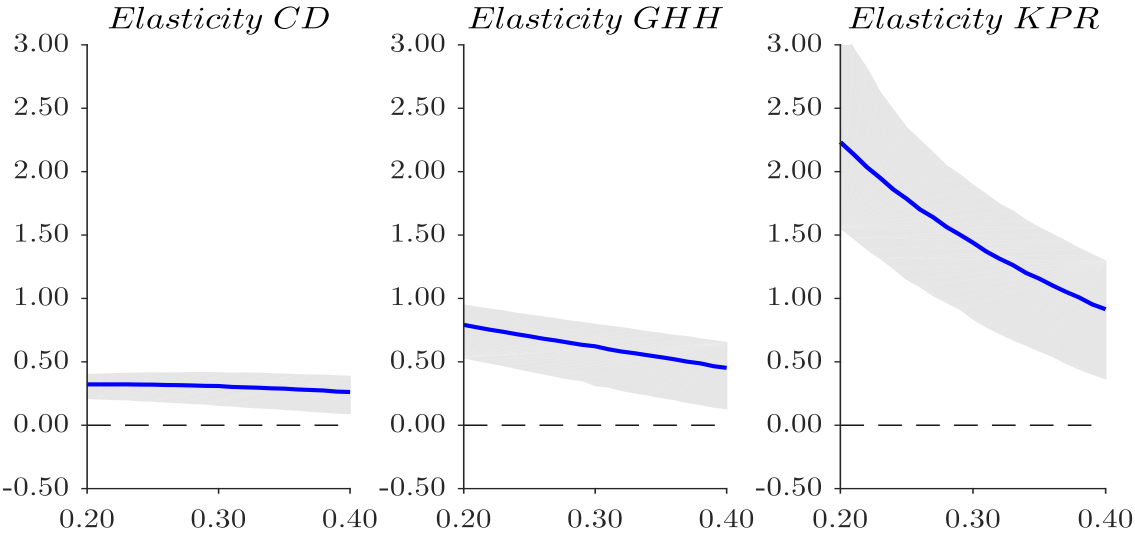
[16] Note that changes in steady state labor L imply different values in steady state consumption C and τ, respectively ψ and ν.

[17] We draw the capital elasticity α from between 0.30 and 0.42, the discount factor β from between 0.90 and 0.995, the depreciation rate δ from 0.01 to 0.05, and the steady state risk aversion σ from 1.50 to 2.50.

FIGURE 3
Euler Equation Elasticities



Note: Figure 3 shows the size of the elasticity of consumption growth with respect to growth in the risk aversion Λ for different steady state values of labor. The blue line is the median and the 5th and 95th percentile confidence bands of 10000 draws for different values of α, β, δ, and σ drawn from a random uniform distribution are shown as the shaded area.

4 Robustness

We subject our model to various alternative specifications and to some variation in the key parameters of the model to verify the robustness of our results.

4.1 Different Model Specifications

Most importantly we test different forms of the risk aversion process in Equation (3). Our main results still hold when we replace the growth rate of final output ΔY_t i.e. of the CES aggregate of home and foreign produced intermediate goods in Equation (6) by the growth rate of the home intermediate good $\Delta Y_{H,t}$ or by the growth rate of GDP i.e. the sum of the home produced intermediate goods for domestic and foreign consumption $Y_{H,t} + Y^*_{H,t}$ in Equation (9). What is crucial in our model is that risk aversion is driven by a country specific process that induces a different degree of risk aversion in each country. Having a risk aversion process that depends on e.g. world output will have no impact on the relative consumption behavior across countries and therefore on the correlation of the real exchange rate and relative consumption.[18] However, since output growth is highly correlated across countries in the model, also the risk aversion shows a high

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[18] Similarly, in a case where the final output good is used but the share of home intermediate goods ω is set to 0.50 our model would fail. However, note that since the law of one price holds for intermediate goods, that in such a case the real exchange rate would be one anyway as the price level is the same in both countries.

degree of cross-country correlation. We also replaced the growth rate of the above mentioned variables by their deviations from the steady state. Although this introduces a slightly different dynamic into the risk aversion, our results still hold and we obtain a near zero correlation of the real exchange rate and relative consumption across countries.

4.2 Different Parameter Values

We now change some parameters of the model that can have a significant impact on cross-country business cycle moments to see whether our model with countercyclical risk aversion is still able to generate moments that are in line with the empirically observed data moments. We do so by changing one parameter from Table 1 at once leaving all other parameters unchanged and then re-estimate our two parameters ρ_σ and η_σ that govern the persistence and degree of countercyclical risk aversion in the model. We opt for showing the second moments for the case of GHH preferences as this kind of preferences are not suffering from the decrease in the correlation of output with consumption and therefore seem to be better suited for an application with time-varying risk aversion as in our model.

We start by using the elasticity of substitution between home and foreign intermediate goods θ of 0.85 as used by Heathcote and Perri (2002). Table 4 shows the second moments for the case of GHH utility when key parameters of the model are changed one by one. As can be seen in column (2), our model is still able to generate a close to zero correlation of the real exchange rate and relative consumption across countries of about 0.10. Compared to the GHH case with a low elasticity of substitution in column (4) of Table 2 our risk aversion parameters ρ_σ and η_σ are virtually unchanged.

In Equation (10) and (11) we allowed for a spillover effect of the TFP shocks to the other country. We now shut down this channel by assuming that ρ_ν^* is 0.00 hence making TFP only dependent on one country's TFP shocks. Column (4) in Table 4 indicates that TFP spillovers between the home and foreign country are not essential for our results. Estimating ρ_σ and η_σ to be 0.51 and 5.11 allows us again to generate a value of -0.02 for the correlation of the real exchange rate and relative consumption.

Finally, we again allow for a spillover in TFP shocks but assume this time that the TFP shocks ν_t and ν_t^* are completely uncorrelated instead of the previously assumed correlation of 0.29. Clearly, column (6) shows that although the cross-correlation in consumption, investment, and labor decreases when we set the TFP shock correlation to zero, we are still able to account for the Backus-Smith Puzzle by generating a negative correlation between the real exchange rate and relative consumption across countries.

TABLE 4
Second Moments - Robustness

		High Elasticity of Substitution		No TFP Spillover		No TFP Correlation	
		(1)	(2)	(3)	(4)	(5)	(6)
	Data	Constant RA	Variable RA	Constant RA	Variable RA	Constant RA	Variable RA
ρ_σ		0.00	0.53	0.00	0.51	0.00	0.19
η_σ		0.00	4.05	0.00	5.11	0.00	7.52
$\sigma(Y)$	1.58	1.94	1.96	1.82	1.86	1.81	1.83
		(1.59;2.40)	(1.58;2.45)	(1.48;2.24)	(1.51;2.32)	(1.46;2.23)	(1.47;2.29)
$\sigma(C)/\sigma(Y)$	0.76	0.85	0.62	0.75	0.55	0.82	0.75
		(0.81;0.90)	(0.54;0.69)	(0.72;0.78)	(0.48;0.60)	(0.77;0.88)	(0.69;0.82)
$\sigma(I)/\sigma(Y)$	4.55	1.56	2.66	1.79	3.26	1.69	3.16
		(1.43;1.70)	(2.43;2.89)	(1.71;1.86)	(3.00;3.53)	(1.53;1.85)	(2.80;3.57)
$\sigma(L)/\sigma(Y)$	0.75	0.59	0.59	0.59	0.60	0.58	0.58
		(0.58;0.60)	(0.58;0.61)	(0.58;0.61)	(0.59;0.61)	(0.56;0.59)	(0.56;0.60)
$\sigma(RER)/\sigma(Y)$	3.06	0.83	1.07	1.60	1.82	1.59	2.02
		(0.66;1.02)	(0.86;1.31)	(1.26;2.00)	(1.43;2.29)	(1.28;1.94)	(1.63;2.45)
$\rho(Y,L)$	0.87	1.00	1.00	1.00	1.00	1.00	1.00
		(1.00;1.00)	(1.00;1.00)	(1.00;1.00)	(1.00;1.00)	(1.00;1.00)	(1.00;1.00)
$\rho(Y,C)$	0.84	0.98	0.81	0.99	0.57	0.97	0.59
		(0.97;0.99)	(0.74;0.86)	(0.98;0.99)	(0.44;0.68)	(0.95;0.98)	(0.42;0.71)
$\rho(Y,I)$	0.91	0.95	0.92	0.98	0.92	0.94	0.83
		(0.92;0.97)	(0.89;0.94)	(0.97;0.99)	(0.90;0.94)	(0.91;0.97)	(0.80;0.86)
$\rho(RER)$	0.82	0.47	0.43	0.60	0.55	0.54	0.44
		(0.33;0.59)	(0.30;0.54)	(0.47;0.71)	(0.42;0.66)	(0.39;0.65)	(0.31;0.56)
$\rho(RER,C/C^*)$	-0.04	0.95	0.10	0.99	-0.02	0.98	-0.28
		(0.94;0.95)	(-0.04;0.25)	(0.99;0.99)	(-0.22;0.18)	(0.97;0.98)	(-0.50;-0.05)
$\rho(Y,Y^*)$	0.44	0.45	0.52	0.67	0.72	0.32	0.39
		(0.19;0.65)	(0.25;0.71)	(0.47;0.80)	(0.53;0.84)	(0.02;0.56)	(0.08;0.62)
$\rho(C,C^*)$	0.36	0.72	0.74	0.78	0.75	0.68	0.55
		(0.56;0.82)	(0.49;0.88)	(0.65;0.87)	(0.50;0.88)	(0.49;0.79)	(0.23;0.73)
$\rho(I,I^*)$	0.28	-0.08	0.28	0.50	0.66	-0.29	0.20
		(-0.38;0.24)	(0.00;0.50)	(0.21;0.70)	(0.48;0.78)	(-0.56;0.03)	(-0.04;0.41)
$\rho(L,L^*)$	0.40	0.54	0.59	0.75	0.78	0.47	0.53
		(0.30;0.71)	(0.35;0.76)	(0.59;0.85)	(0.63;0.88)	(0.20;0.67)	(0.26;0.72)

Note: Table 2 shows the moments of the model for GHH preferences. σ denotes the standard deviation of a variable and ρ denotes the correlation between two variables. 5th and 95th percentile confidence bands in parenthesis. Moments are the median of 500 replications.

5 Conclusion

There is increasing empirical evidence that the risk aversion of agents is time-varying and countercyclical. In this paper we introduced countercyclical risk aversion into a standard international RBC model and analyzed its effects on international business cycle moments. Once the risk aversion becomes slightly countercyclical the correlation between the real exchange rate and relative consumption across countries vanishes and turns zero or slightly negative as observed in the data. The model is hence able to account for the Backus-Smith Puzzle. In addition our model induces a higher cross-country correlation of investment and labor and therefore makes some progress in the International-Comovement Puzzle. Based on the behavioral findings by Cohn et al. (2015) and Guiso et al. (2018) we introduced countercyclical risk aversion in a reduced form process. We documented that our findings are robust to various functional forms of the preferences. Further research might want to be more explicit about how agents' risk aversion is affected by macroeconomic fundamentals e.g. by introducing a learning process for the risk aversion.

A Appendix

A.1 Steady State

We here derive the steady states for the home economy. Since both economies are fully symmetric the steady states for the home and the foreign economy are identical. We know that in the steady state

$$Q = \beta. \tag{A1}$$

The output to capital ratio can then be derived knowing that the marginal product of capital equals $\frac{1}{Q} - (1 - \delta)$

$$\frac{Y}{K} = \frac{\frac{1}{Q} - (1 - \delta)}{\alpha}. \tag{A2}$$

From the budget constraint we get by knowing that in steady state $D = 0$ and that the labor and wage share add up to one

$$\frac{C}{Y} = 1 - \delta \frac{K}{Y} \tag{A3}$$

where $\frac{K}{Y}$ is the inverse of the output to capital ratio in Equation (A2). Steady state labor is exogenously fixed at

$$L = 0.30. \tag{A4}$$

Capital follows from the Cobb-Douglas production function as

$$K = \left[\frac{L^{(1-\alpha)}}{\frac{Y}{K}} \right]^{\frac{1}{(1-\alpha)}} \tag{A5}$$

where $\frac{Y}{K}$ is the output to capital ratio from Equation (A2). Steady state output is produced using a Cobb-Douglas function and follows as

$$Y = K^{\alpha} L^{1-\alpha}. \tag{A6}$$

Consumption can be derived from the consumption to output ratio in Equation (A3) and output in Equation (A6)

$$C = \frac{C}{Y} Y. \tag{A7}$$

Investment in the steady state follows from the capital law of motion as

$$I = \delta K. \tag{A8}$$

The return on capital equals

$$R = \frac{1}{Q} - (1 - \delta). \tag{A9}$$

Wages are the marginal product of labor

$$W = (1 - \alpha) K^{\alpha} L^{-\alpha} \tag{A10}$$

where K is steady state capital as in Equation (A5) and L is the exogenously fixed steady state labor supply from Equation (A4). The home intermediate good is

$$Y_H = \omega Y \tag{A11}$$

where ω is the share of home intermediate goods. The foreign intermediate good is then

$$Y_F = (1 - \omega) Y. \tag{A12}$$

For the foreign country we have that in steady state

$$Y_H^* = (1 - \omega) Y \tag{A13}$$

and

$$Y_F^* = \omega Y. \tag{A14}$$

Since in the steady state $\Delta Y = 0$ the AR(1) process for the risk aversion becomes

$$\gamma = 0 \tag{A15}$$

and the risk aversion in steady state becomes

$$\sigma = 2. \tag{A16}$$

We finally know that in steady state the price of the final good P, the price of the home intermediate good P_H, and the price of the foreign intermediate good P_F are 1 as well as the real exchange rate $RER = 1$. Table A1 provides the steady states for the parameter values shown in Table 1. Since both countries are symmetric, the steady states are the same for both countries.

TABLE A1
Steady States

Variable	Description	Steady State
Y	Output	1.11
Y_H	Home Intermediate Good	1.00
Y_F	Foreign Intermediate Good	0.11
C	Consumption	0.83
I	Investment	0.28
K	Capital	11.40
L	Labor	0.30
R	Return on Capital	0.035
W	Wage	2.37
D	Debt	0.00
Y/K	Output to Capital Ratio	0.10
C/Y	Consumption to Output Ratio	0.74
γ	Risk Aversion Process	0.00
σ	Risk Aversion	2.00
Q	Debt Price	0.98

Note: Table A1 shows the steady states of the model using the parameters in Table 1.

A.2 Equilibrium Conditions

We give here the full set of equations to solve the model. Relative prices of home and foreign intermediate goods are defined as

$$\tilde{P}_{H,t} = \frac{P_{H,t}}{P_t} \tag{A17}$$

$$\tilde{P}^*_{F,t} = \frac{P_{F,t}}{P^*_t} \tag{A18}$$

Final good prices using that the law of one price holds so that $P_{H,t} = P^*_{H,t}$ and $P_{F,t} = P^*_{F,t}$

$$P_t = \left[\omega P_{H,t}^{1-\theta} + (1-\omega) P_{F,t}^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{A19}$$

$$P^*_t = \left[\omega P_{F,t}^{1-\theta} + (1-\omega) P_{H,t}^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{A20}$$

Labor supply

$$-\frac{U_{t,L}}{U_{t,C}} = W_t \tag{A21}$$

$$-\frac{U^*_{t,L}}{U^*_{t,C}} = W^*_t \tag{A22}$$

Marginal utilities for labor and consumption in the Cobb-Douglas, GHH, and KPR case are defined in the main text in Equation (17) and (18) for Cobb-Douglas preferences, in Equation (21) and (22) for GHH preferences, and in Equation (25) and (26) for KPR preferences. Euler equations for investment with λ_t and μ_t being the Lagrange multiplier of the budget constraint and the capital law of motion

$$\lambda_t = \mu_t \left[1 - \phi \left(\frac{I_t}{I_{t-1}} - 1\right)\right] + \beta \mathbb{E}_t \mu_{t+1} \left[\phi \left(\frac{I_{t+1}}{I_t} - 1\right) \frac{I_{t+1}}{I_t} - \frac{\phi}{2} \left(\frac{I_{t+1}}{I_t} - 1\right)^2\right] \tag{A23}$$

$$\lambda_t^* = \mu_t^* \left[1 - \phi \left(\frac{I_t^*}{I_{t-1}^*} - 1\right)\right] + \beta \mathbb{E}_t \mu_{t+1}^* \left[\phi \left(\frac{I_{t+1}^*}{I_t^*} - 1\right) \frac{I_{t+1}^*}{I_t^*} - \frac{\phi}{2} \left(\frac{I_{t+1}^*}{I_t^*} - 1\right)^2\right] \tag{A24}$$

Capital Euler

$$\mu_t = \beta \mathbb{E}_t \left[R_{t+1} \lambda_{t+1} + \mu_{t+1} \left(1 - \delta\right)\right] \tag{A25}$$

$$\mu_t^* = \beta \mathbb{E}_t \left[R_{t+1}^* \lambda_{t+1}^* + \mu_{t+1}^* \left(1 - \delta\right)\right] \tag{A26}$$

Capital law of motion

$$K_t = \left(1 - \delta\right) K_{t-1} + I_t - \frac{\phi}{2} I_{t-1} \left(\frac{I_t}{I_{t-1}} - 1\right)^2 \tag{A27}$$

$$K_t^* = \left(1 - \delta\right) K_{t-1}^* + I_t^* - \frac{\phi}{2} I_{t-1}^* \left(\frac{I_t^*}{I_{t-1}^*} - 1\right)^2 \tag{A28}$$

Risk sharing where the real exchange rate $RER_t = \frac{P_t^*}{P_t}$

$$\mathbb{E}_t \left[\frac{U_{t+1,C}^*}{U_{t,C}^*} \frac{\tilde{P}_{H,t+1}}{\tilde{P}_{H,t}} \frac{RER_t}{RER_{t+1}}\right] = \mathbb{E}_t \left[\frac{U_{t+1,C}}{U_{t,C}} \frac{\tilde{P}_{H,t+1}}{\tilde{P}_{H,t}}\right] - \frac{\zeta}{\beta} D_t \tag{A29}$$

Consumption Euler

$$\frac{Q_t + \zeta D_t}{\beta} = \mathbb{E}_t \left[\frac{U_{t+1,C}}{U_{t,C}} \frac{\tilde{P}_{H,t+1}}{\tilde{P}_{H,t}}\right] \tag{A30}$$

Wages and capital returns derived from the intermediate goods producers optimization problem

$$R_t = \alpha e^{A_t} \tilde{P}_{H,t} K_{t-1}^{\alpha-1} L_t^{1-\alpha} \tag{A31}$$

$$R_t^* = \alpha e^{A_t^*} \tilde{P}_{F,t}^* \left(K_{t-1}^*\right)^{\alpha-1} \left(L_t^*\right)^{1-\alpha} \tag{A32}$$

$$W_t = \left(1 - \alpha\right) e^{A_t} \tilde{P}_{H,t} K_{t-1}^{\alpha} L_t^{-\alpha} \tag{A33}$$

$$W_t^* = \left(1 - \alpha\right) e^{A_t^*} \tilde{P}_{F,t}^* \left(K_{t-1}^*\right)^{\alpha} \left(L_t^*\right)^{-\alpha} \tag{A34}$$

Demand for intermediate goods derived from the final goods producers optimization problem

$$Y_{H,t} = \omega \tilde{P}_{H,t}^{-\theta} Y_t \tag{A35}$$

$$Y_{F,t} = \left(1 - \omega\right) \left(\tilde{P}_{F,t} RER_t\right)^{-\theta} Y_t \tag{A36}$$

$$Y_{H,t}^{*} = (1-\omega)\left(\frac{\tilde{P}_{H,t}}{RER_t}\right)^{-\theta} Y_t^{*} \tag{A37}$$

$$Y_{F,t}^{*} = \omega \tilde{P}_{F,t}^{-\theta} Y_t^{*} \tag{A38}$$

Market clearing final good

$$Y_t = C_t + I_t \tag{A39}$$

$$Y_t^{*} = C_t^{*} + I_t^{*} \tag{A40}$$

Market clearing intermediate good

$$Y_{H,t} + Y_{H,t}^{*} = e^{A_t} K_{t-1}^{\alpha} L_t^{1-\alpha} \tag{A41}$$

$$Y_{F,t} + Y_{F,t}^{*} = e^{A_t^{*}} \left(K_{t-1}^{*}\right)^{\alpha} \left(L_t^{*}\right)^{1-\alpha} \tag{A42}$$

Final good production

$$Y_t = \left[\omega^{\frac{1}{\theta}} Y_{H,t}^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} Y_{F,t}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{A43}$$

$$Y_t^{*} = \left[\omega^{\frac{1}{\theta}} \left(Y_{F,t}^{*}\right)^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} \left(Y_{H,t}^{*}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{A44}$$

Processes for TFP

$$A_t = \rho_A A_{t-1} + \rho_A^{*} A_{t-1}^{*} + u_t \tag{A45}$$

$$A_t^{*} = \rho_A A_{t-1}^{*} + \rho_A^{*} A_{t-1} + u_t^{*} \tag{A46}$$

Risk aversion

$$\sigma_t = \sigma + e^{\gamma_t} - 1 \tag{A47}$$

$$\sigma_t^{*} = \sigma + e^{\gamma_t^{*}} - 1 \tag{A48}$$

Risk aversion processes

$$\gamma_t = \rho_\sigma \gamma_{t-1} - \eta_\sigma \Delta Y_t \tag{A49}$$

$$\gamma_t^{*} = \rho_\sigma \gamma_{t-1}^{*} - \eta_\sigma \Delta Y_t^{*} \tag{A50}$$

Growth rate of final good

$$\Delta Y_t = \frac{Y_t - Y_{t-1}}{Y_{t-1}} \tag{A51}$$

$$\Delta Y_t^{*} = \frac{Y_t^{*} - Y_{t-1}^{*}}{Y_{t-1}^{*}} \tag{A52}$$

Law of motion of the bond

$$P_{H,t}Q_tD_t = P_{H,t}Y^*_{H,t} - P_{F,t}Y_{F,t} + P_{H,t}D_{t-1} - P_{H,t}\frac{\zeta}{2}D_t^2 \tag{A53}$$

A.3 Log-Linear Euler Equations

A.3.1 GHH

The Euler equation with GHH preferences becomes

$$\frac{Q_t + \zeta D_t}{\beta} = \mathbb{E}_t\left[\frac{(C_t - \psi L_t^{\nu})^{\sigma_t}}{\left(C_{t+1} - \psi L_{t+1}^{\nu}\right)^{\sigma_{t+1}}}\frac{\tilde{P}_{H,t+1}}{\tilde{P}_{H,t}}\right] \tag{A54}$$

or in logarithmic terms

$$\ln\left(Q_t + \zeta D_t\right) - \ln\left(\beta\right) + \ln\left(\tilde{P}_{H,t}\right) - \ln\left(\tilde{P}_{H,t+1}\right) = \sigma_t \ln\left(C_t - \psi L_t^{\nu}\right) - \sigma_{t+1}\ln\left(C_{t+1} - \psi L_{t+1}^{\nu}\right). \tag{A55}$$

Log-linearizing around the steady state gives

$$\begin{aligned}-\widetilde{r_t} + \zeta\widetilde{D_t} + \widetilde{\tilde{P}_{H,t}} - \widetilde{\tilde{P}_{H,t+1}} = \frac{\sigma C}{C - \psi L^{\nu}}\widetilde{C_t} - \frac{\sigma C}{C - \psi L^{\nu}}\widetilde{C_{t+1}} - \frac{\sigma\psi\nu L^{\nu}}{C - \psi L^{\nu}}\widetilde{L_t} + \frac{\sigma\psi\nu L^{\nu}}{C - \psi L^{\nu}}\widetilde{L_{t+1}} \\ + \ln\left(C - \psi L^{\nu}\right)\sigma\widetilde{\sigma_t} - \ln\left(C - \psi L^{\nu}\right)\sigma\widetilde{\sigma_{t+1}}.\end{aligned} \tag{A56}$$

Then after rearranging terms we get for consumption growth

$$\widetilde{C_{t+1}} - \widetilde{C_t} = \left(\widetilde{r_t} - \zeta\widetilde{D_t} - \widetilde{\tilde{P}_{H,t+1}} + \widetilde{\tilde{P}_{H,t}}\right)\frac{C - \psi L^{\nu}}{\sigma C} + \Gamma\left(\widetilde{L_{t+1}} - \widetilde{L_t}\right) + \Lambda\left(\widetilde{\sigma_{t+1}} - \widetilde{\sigma_t}\right) \tag{A57}$$

where

$$\Gamma = \frac{\psi\nu L^{\nu}}{C} \tag{A58}$$

and

$$\Lambda = -\ln\left(C - \psi L^{\nu}\right)\frac{C - \psi L^{\nu}}{C}. \tag{A59}$$

A.3.2 KPR

The Euler equation with KPR preferences becomes

$$\frac{Q_t + \zeta D_t}{\beta} = \mathbb{E}_t\left[\frac{\left[C_t\left(1 - \psi L_t^{\nu}\right)\right]^{\sigma_t}\left(1 - \psi L_{t+1}^{\nu}\right)}{\left[C_{t+1}\left(1 - \psi L_{t+1}^{\nu}\right)\right]^{\sigma_{t+1}}\left(1 - \psi L_t^{\nu}\right)}\frac{\tilde{P}_{H,t+1}}{\tilde{P}_{H,t}}\right] \tag{A60}$$

or in logarithmic terms

$$\ln\left(Q_t + \zeta D_t\right) - \ln\left(\beta\right) + \ln\left(\tilde{P}_{H,t}\right) - \ln\left(\tilde{P}_{H,t+1}\right) = \sigma_t \ln\left(C_t\right) + \sigma_t \ln\left(1 - \psi L_t^{\nu}\right)$$

$$+\ln\left(1-\psi L_{t+1}^{\nu}\right)-\sigma_{t+1}\ln\left(C_{t+1}\right)-\sigma_{t+1}\ln\left(1-\psi L_{t+1}^{\nu}\right)-\ln\left(1-\psi L_{t}^{\nu}\right). \quad \text{(A61)}$$

Log-linearizing around the steady state gives

$$\begin{aligned}-\widetilde{r_t}+\zeta\widetilde{D_t}+\widetilde{\tilde{P}_{H,t}}-\widetilde{\tilde{P}_{H,t+1}}=\sigma\widetilde{C_t}-\sigma\widetilde{C_{t+1}}+\widetilde{L_t}\left[\frac{\psi\nu L^{\nu}-\sigma\psi\nu L^{\nu}}{1-\psi L^{\nu}}\right]-\widetilde{L_{t+1}}\left[\frac{\psi\nu L^{\nu}-\sigma\psi\nu L^{\nu}}{1-\psi L^{\nu}}\right]\\+\sigma\widetilde{\sigma_t}\left[\ln\left(C\right)+\ln\left(1-\psi L^{\nu}\right)\right]-\sigma\widetilde{\sigma_{t+1}}\left[\ln\left(C\right)+\ln\left(1-\psi L^{\nu}\right)\right]. \quad \text{(A62)}\end{aligned}$$

Then after rearranging terms we get for consumption growth

$$\widetilde{C_{t+1}}-\widetilde{C_t}=\frac{\widetilde{r_t}-\zeta\widetilde{D_t}-\widetilde{\tilde{P}_{H,t+1}}+\widetilde{\tilde{P}_{H,t}}}{\sigma}+\Gamma\left(\widetilde{L_{t+1}}-\widetilde{L_t}\right)+\Lambda\left(\widetilde{\sigma_{t+1}}-\widetilde{\sigma_t}\right) \quad \text{(A63)}$$

where

$$\Gamma=\frac{\sigma-1}{\sigma}\left[\frac{\psi\nu L^{\nu}}{1-\psi L^{\nu}}\right] \quad \text{(A64)}$$

and

$$\Lambda=-\left[\ln\left(C\right)+\ln\left(1-\psi L^{\nu}\right)\right]. \quad \text{(A65)}$$

A.4 Derivation of the Cobb-Douglas Consumption Share

Combining the labor supply equation in Equation (A21) and the steady state expression for wages in Equation (A10) gives

$$\left(\frac{1-\tau}{1-L}\right)\frac{C}{\tau}=\left(1-\alpha\right)K^{\alpha}L^{-\alpha} \quad \text{(A66)}$$

dividing both sides by steady state production Y and substituting in

$$\left(\frac{1-\tau}{1-L}\right)\frac{C}{Y}\frac{1}{\tau}=\frac{\left(1-\alpha\right)K^{\alpha}L^{-\alpha}}{K^{\alpha}L^{1-\alpha}} \quad \text{(A67)}$$

so that canceling terms and multiplying both sides by $(1-L)$ gives

$$\left(\frac{1-\tau}{\tau}\right)\frac{C}{Y}=\frac{\left(1-L\right)\left(1-\alpha\right)}{L} \quad \text{(A68)}$$

so that when multiplying by Y, dividing by C, and subsequently adding one on both sides yields

$$\frac{1}{\tau}=\frac{CL}{CL}+\frac{Y\left(1-L\right)\left(1-\alpha\right)}{CL} \quad \text{(A69)}$$

and by taking the reciprocal one gets

$$\tau = \frac{CL}{CL + Y\left(1 - L\right)\left(1 - \alpha\right)} \tag{A70}$$

multiplying and dividing the right hand side by Y then gives

$$\tau = \frac{L\frac{C}{Y}}{L\frac{C}{Y} + \left(1 - L\right)\left(1 - \alpha\right)}. \tag{A71}$$

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